CAN ONE HEAR THE SHAPE OF A ROOM: THE 2-D POLYGONAL CASE

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ABSTRACT

We consider the problem of estimating room geometry from the acoustic room impulse response (RIR). Existing approaches addressing this problem exploit the knowledge of multiple RIRs. In contrast, we are interested in reconstructing the room geometry from a single RIR — a 1–D function of time. We discuss the uniqueness of the mapping between the geometry of a planar polygonal room and a single RIR. In addition to this theoretical analysis, we also propose an algorithm that performs the "blindfolded" room estimation. Furthermore, the derived results are used to construct an algorithm for localization in a known room using only a single RIR. Verification of the theoretical developments with numerical simulations is given before concluding the paper.

Index Terms— Drum shape, room impulse response, room geometry estimation, room acoustics, image source model

1. INTRODUCTION

In a famous paper [1], M. Kac asks the catchy question "Can you hear the shape of a drum?". This problem is related to a question in astrophysics, and the answer is negative, meaning, different drum shapes can have the same resonant frequencies. In this paper, we ask the same question, but for the acoustic room impulse response (RIR). That is, assume you are blindfolded inside a room, you snap your fingers and you listen to the impulse response. Can you hear the shape of the room? Intuitively, and for simple shapes, we know this to be true: A rectangular room, for example, has well defined modes, from which we can derive its size. But the question is challenging in more general cases, even if we feel that the RIR contains an arbitrarily long set of echoes (assuming an ideal, noiseless measurement) which ultimately should specify the geometry of the room.

Beyond the question of uniqueness, meaning the RIR being a unique signature of a room, the question of reconstructing the geometry from the RIR is an interesting algorithmic question. Finally, uniqueness will lead to localization inside a known room and algorithms for tracking the trajectory of a moving source listening to the varying RIRs.

1.1. Prior Art

Recently, there has been renewed interest in reconstructing the room shape from acoustic response, as shown by several papers at ICASSP-2010. To the best of our knowledge, all the papers deal with array processing methods. In [2] the authors propose to circle a loudspeaker around a microphone to collect multiple impulse

responses and then to estimate the distance and the angle of the reflector (a line since they consider a 2-D case) using the tools of projective geometry. They take into account first order reflections and choose not to discuss the assignment of delayed pulses to specific walls. An ℓ_1 regularized least squares method is used in [3] to estimate the geometry of a shoebox room by measuring many single wall impulse responses under different angles. A different approach is proposed in [4] where the authors do not assume an *a priori* knowledge of the excitation signal, therefore not assuming the knowledge of the impulse response either. An approach based on acoustic imaging is proposed in [5]. The authors use a microphone array to sample the sound field and then employ wave field inversion to infer the room.

1.2. Paper Outline and Main Contributions

In Section 2 we describe our setup and the adopted image source model. We also discuss the implications of the model in the statistical characterization of RIRs. The uniqueness of RIR in a convex polygonal room (up to some symmetries) is shown in Section 3. In Section 3 we also give an algorithm to recover the shape of the room based on its RIR. In Section 4 we solve the problem of localization in a room based on the RIR, including the tracking of a trajectory. Numerical simulations presented in Section 5 confirm the effectiveness of the proposed algorithms.

We omit the discussion of delay estimation as it is outside of the primarily theoretical scope of this paper. Also, due to space limitations, we leave the proofs of some lemmas and theorems to a forth-coming extended version of this paper. Finally, for simplicity of exposition, we only consider 2–D polygonal rooms in this paper. All the derivations and results can be easily extended to the 3–D case.

2. PROBLEM SETUP

We consider a setup that consists of a sound source and a colocated microphone, both omnidirectional. Assume that an omnidirectional pulse is emitted from somewhere inside the room, and that the room response is collected at the same point. From the collected RIR and the knowledge of the emitted pulse, we can extract the set of delays. In all derivations, our choice of units is such that the speed of sound is unity.

2.1. Image Source Model

In order to model room acoustics we adopt the image source model. Common references are the work of Allen and Berkley [6] for shoebox rooms and an extension to general polyhedra in [7]. The idea here is that if there is a sound source on one side of the wall, then the sound field on the same side can be represented as a superposition

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Fig. 1. Setup with colocated source and receiver. Source is assumed to be at the origin. p_i and p_{i+1} are endpoints of *i*th wall, n_i is its unit, outward pointing normal, c_i is the center of the line segment $p_i p_{i+1}$, and q_i is the first generation image source. Its image with respect to the (i + 1)st wall is q_{ii} .

of the original sound field and the one generated by a mirror image of the source with respect to the wall. Fig. 1 illustrates the setup and the image source model.

For the purpose of this paper, a room is a convex planar K-polygon represented by a $2 \times K$ vertex matrix $P = [p_1, \dots, p_K]$. We assume that the vertices are specified in a counterclockwise direction and define the *i*th side of the room as the line segment joining p_i and p_{i+1} . Without loss of generality we place our source at the origin and require that the room contains the origin. Since the source and the listener are colocated, it is not possible to discriminate rotated and reflected variants of a room about the source. Therefore, we think of these as being the same room. We can resolve this ambiguity by choosing (fixing) some degrees of freedom, e.g., if *i*th side is closest to the source, we set it to be vertical and choose that the closer of the adjacent sides follows in a CCW direction.

With the *i*th side of the room we associate an outward pointing unit normal n_i , and define the normal matrix as $N \stackrel{\text{def}}{=} [n_1, \ldots, n_K]$. We denote by q_i the image source location with respect to the *i*th side. The set $\{q_i\}_{1 \le i \le K}$ contains what we call first generation image sources. Analogously, the image of the virtual source q_i with respect to the wall *j* is denoted q_{ij} and the $\{q_{ij}\}_{1 \le i \ne j \le K}$ is the set of second generation image sources.

By observing the impulse response we have access to $G_1 = \{ \| \boldsymbol{q}_i \| \}$ and to $G_2 = \{ \| \boldsymbol{q}_{ij} \| \}$, sets of first and second generation delays.

2.2. Characterization of RIRs by Looking at the Image Sources

Fig. 2 shows the image source patterns for different rooms. Looking at the generated image source patterns reveals several known facts in room acoustics in an intuitive way. One can observe the difference in the behaviour of *regular* and *irregular* rooms.

For rectangular rooms the pattern of image sources corresponds to a union of four lattices in \mathbb{R}^2 . A lattice $\Lambda_M \in \mathbb{R}^2$ generated by matrix $M \in \mathbb{R}^{2\times 2}$ is defined as $\Lambda_M = \{x | x = Mn, n \in \mathbb{Z}^2\}$. From Fig. 2a) and Fig. 2b) we can observe that other regular polygons also generate regular image source patterns that correspond to unions of lattices. In contrast, Fig. 2c) shows that the image source pattern generated by a *random* triangle is not regular at all. In fact, an interesting effect is observed — as we move away from the original source, the density of virtual sources increases. From this pattern we observe an interesting scaling law for regular rooms. The density of pulses in the RIR is growing with time as t^{D-1} where D is the dimensionality of the ambient space, e.g., constant for 1-D rooms, $\sim t$ in 2–D rooms and $\sim t^2$ in 3–D rooms. For a 2–D room, this means that the number of image sources inside the ring of constant width grows linearly with the radius of the ring.

If we consider the irregular triangle in Fig. 2c) it becomes clear that the same scaling statement does not hold. This provides an intuitive explanation of why oblique walls exhibit specific acoustic properties.

By examining the image source pattern generated by a regular hexagon, one can distinguish between the sources corresponding to discrete early reflections, and the far sources that will generate the diffuse reverberation — a known behavior from room acoustics. A sampled RIR with these annotations is given in Fig. 2d).

These examples suggest a very strong link between the room geometry — what we want to know — and the corresponding impulse response — what we hear. We show that under right conditions this link is an invertible mapping.

3. ROOM GEOMETRY ESTIMATION

In this section we derive the mapping between the room geometry and the RIR. We also discuss its uniqueness, and give an algorithm to retrieve the room geometry from the measured RIR.

3.1. The Shape of a Polygonal Room Using Matrix Analysis

It is not possible to reconstruct the room geometry using only the G_1 delays. To see this, consider a triangle with the corresponding set of first generation delays. Now choose one side and tilt it so that you change the shape of the triangle. This can only change one delay in G_1 . But now we can translate this side keeping all the angles fixed until we match this changed delay with the old one, ending up with two rooms with the same G_1 .

We claim however, that G_1 and G_2 delays are sufficient to recover the room in a large number of cases. First we set up the link between the geometry of the room and the measured RIR (G_1 and G_2).

Lemma 1. Let the room vertices be given in P. Associate with this room a matrix $A = \text{diag}(a_1, \ldots, a_K)$ (diagonal matrix having $a_i = ||\mathbf{q}_i||$ as the ith diagonal entry), and a matrix $\mathbf{Q} = (||\mathbf{q}_{ij}||^2)$, having the second order delays as its elements. Furthermore, let E = ones(K) be a $K \times K$ matrix of ones, and N be a matrix of normals corresponding to P. Then the following holds,

$$N^T N = A^{-1} (A^2 E + E A^2 - Q) A^{-1} / 2.$$
 (1)

Proof. From Fig. 1 we obtain that

$$\boldsymbol{q}_i = 2\langle \boldsymbol{c}_i, \boldsymbol{n}_i \rangle \boldsymbol{n}_i, \tag{2}$$

where c_i is the midpoint of *i*th side. In the second generation we consider each of the K first-generation virtual sources as the new source and using the same logic as above compute the second-generation virtual sources. Since in (2) we assumed the source to be at the origin, now we move the origin to q_i ,

$$\begin{aligned} \boldsymbol{q}_{ij} &= \boldsymbol{q}_i + 2\langle \boldsymbol{c}_j - \boldsymbol{q}_i, \boldsymbol{n}_j \rangle \boldsymbol{n}_j \\ &= \boldsymbol{q}_i + \boldsymbol{q}_j - 2\langle \boldsymbol{q}_i, \boldsymbol{n}_j \rangle \boldsymbol{n}_j. \end{aligned} \tag{3}$$



Fig. 2. Image source patterns for different rooms. (a) equilateral triangle, regular pattern, (b) regular hexagon, again regular pattern, (c) random triangle with 25 generations of virtual sources, irregular pattern. Since the patterns in (a) and (b) have *constant density*, the time density of the corresponding echoes scales as $\sim t$. (d) simulated RIR (plot in (d) was generated using the code described in [8]).

Let $a_i \stackrel{\text{def}}{=} 2\langle \boldsymbol{c}_i, \boldsymbol{n}_i \rangle = \|\boldsymbol{q}_i\|$ and $n_{ij} \stackrel{\text{def}}{=} \langle \boldsymbol{n}_i, \boldsymbol{n}_j \rangle$. Then using (2) and (3) we obtain

$$\|\boldsymbol{q}_{ij}\|^2 = a_i^2 + a_j^2 - 2a_i a_j n_{ij}.$$
(4)

This gives us G_2 in terms of G_1 and an inner product between corresponding normals. A particular consequence of (4) is that $||q_{ij}|| = ||q_{ji}||$. This means that in the second generation we can resolve at most K(K-1)/2 distinct pulses in the impulse response.

But with some simple manipulations (4) can be stated in a matrix form as

$$\boldsymbol{Q} = \boldsymbol{A}^2 \boldsymbol{E} + \boldsymbol{E} \boldsymbol{A}^2 - 2\boldsymbol{A} \boldsymbol{N}^T \boldsymbol{N} \boldsymbol{A}.$$
 (5)

Now the claim of the lemma follows directly.

The norm equation (4) is due to the straight line geometry of our problem. In a work on transient light imaging [9], the authors use a ray model and obtain a similar equation for pairwise reflector distances.

Thus we get a simple expression that links the room geometry with delay times in the impulse response. Notice that a_i is the *i*th first generation delay, so by measuring the RIR we get access to both A and Q. This means that we can easily solve for $N^T N$. By applying the SVD to this matrix, we get N, the matrix of normals. But A and N completely determine the room shape, so estimating the room geometry becomes equivalent to an SVD computation. There is a catch however: even if we know G_1 and G_2 , we do not know how to order them in A and Q, so we end up with an assignment problem. Before discussing it we state a useful consequence of Lemma 1.

Corollary 1. If Q is defined as in the statement of Lemma 1 then rank $Q \leq 4$.

Proof. Since $N \in \mathbb{R}^{2 \times K}$ and A is of a full rank, rank $(N^T N)$ = rank $(AN^T NA)$ = 2. Also, it is not hard to see that rank $(A^2 E)$ = rank (EA^2) = 1. So rank $(Q) \leq \operatorname{rank}(A^2 E)$ + rank (EA^2) + rank $(N^T N)$ = 4 by rank inequalities.

3.2. Uniqueness of RIR: Assignment Problem

In this section we show that under the right circumstances it is possible to correctly assign the G_1 and G_2 delays to matrices A and Q and therefore recover the room using (1).

Definition 1. We say that a room is feasible if it allows the listener at the origin to hear K echoes in G_1 and K(K-1)/2 echoes in G_2 , when an omnidirectional pulse is emitted from the origin.

In practice, this means that the source and the listener should be inside the feasible region (e.g. avoid getting too close to corners). To prepare the ground for the main result, we put forward some properties of the involved matrices.

Consider a K-room with vertices P and corresponding normals N. Let A and Q be the correctly permuted matrices corresponding to the room P. Furthermore, let π be the permutation operator that acts on matrices, in such a way that if R is some matrix related to the room P then $\pi(R)$ is the matrix that we would get if we relabeled the vertices in P according to π . Then the following lemma holds.

Lemma 2. Let
$$A_{\pi} = \pi(A), Q_{\pi} = \pi(Q)$$
 and $N_{\pi}^{T}N_{\pi} = \pi(N^{T}N)$. Then $N_{\pi}^{T}N_{\pi} = A_{\pi}^{-1}(A_{\pi}^{2}E + EA_{\pi}^{2} - Q_{\pi})A_{\pi}^{-1}/2$.

In words, we do not have to search for an absolutely correct arrangement of delays. We only have to match the permutation of Q and A. If we plug these into (1) we obtain the permuted N. By sorting the normals in a CCW direction we find the correct room. Notice that rank $(N_{\pi}^{T}N_{\pi}) = 2$.

Now we need a way to tell if a *relative* arrangement between A and Q is wrong. This is formalized in the following lemma.

Lemma 3. (Detectability) Let
$$A_{\pi_1} = \pi_1(A), Q_{\pi_2} = \pi_2(Q)$$
 where $\pi_1 \neq \pi_2$. Then rank $(A_{\pi_1}^{-1}(A_{\pi_1}^2 E + EA_{\pi_1}^2 - Q_{\pi_2})A_{\pi_1}^{-1}/2) > 2$.

This means that we have a tool for detecting the wrong relative arrangement: just plug A_{π_1} and Q_{π_2} into (1) and check the rank. Collecting these results, we are in a position to state the following:

Theorem 1. Among all feasible rooms there is exactly one room that generates given G_1 , G_2 (given that these are generated by a feasible room). This room can be retrieved by Algorithm 1.

Algorithm 1 along with the above lemmas gives a constructive proof of the recovery of the unique room (up to rotation and reflection) from G_1 and G_2 .

4. EXTENSION: INDOOR LOCALIZATION

How can we use the results of the previous section to localize a source inside a *known* room? Apparently, geometry estimation algorithm also gives the listener location, so the question is what is the difference if we know the room geometry? Localization with a known room geometry means that each wall has a definite *label* so the location should be given in terms of these labels (e.g., 3 meters)

Algorithm 1 Room recovery

- (i) Combine the delays from G_2 in Q_{π} until rank $(Q_{\pi}) \leq 4$,
- (ii) Rearrange the diagonal of A_{π} until it is matched with computed Q_{π} , i.e., rank $(A_{\pi}^2 E + E A_{\pi}^2 - Q_{\pi}) = 2$. If this happens for no A_{π} , repeat from (i),
- (iii) Apply SVD to $N_{\pi}^T N_{\pi} = A_{\pi}^{-1} (A_{\pi}^2 E + E A_{\pi}^2 Q_{\pi}) A_{\pi}^{-1} / 2$ to get N_{π}
- (iv) Sort $n_{\pi,i}$'s and $a_{\pi,i}$'s in a CCW direction and intersect to get the polygon P.

from wall A, 4 meters from wall B, ...). This fixed labeling forces the ordering of elements in all the involved matrices.

By knowing the room geometry, we know N. Furthermore, we observe G_1 and G_2 . By the results of the previous section, if we plug $N^T N$ and A into (5) we should get Q with the observed G_2 delays. But if we use the wrong permutation of A we end up with a wrong Q.

This leads to Algorithm 2. We do not completely avoid the combinatorial search but we might be able to run this search only once in a while, if we consider the location updating scenario. This should be possible if the displacement between two runs of the localization algorithm is not large, so that the assignment does not change between two measurements. It is easy to detect that the assignment changed, and only then we should rerun the combinatorial search.

Algorithm 2 Localize and update

- (i) Rearrange A until $A^2 E + EA^2 2AN^T NA$ matches the observed G_2 (in a noisy scenario, we look for A minimizing the ℓ_2 distance between the delays in G_2 and Q),
- (ii) To update the location, use the arrangement of *A* from (i) until *Q* becomes corrupt (since the source speed is finite we can discard all *A*s that yield an overly large displacement),
- (iii) When Q becomes corrupt, repeat (i) (start with cycles, i.e. just try swapping two sides, as the displacement is still small).

5. NUMERICAL SIMULATIONS

We have validated the theoretical results on a number of numerical simulations, but for reasons of space here we only give two examples. To simulate the uncertainties in the timing estimation, we add Gaussian noise to the simulated delay times and feed them into the proposed algorithms.

Fig. 3a) shows geometry estimation for a quadrilateral room. Green line shows the estimated room in noiseless conditions and is identical to the actual room to within numerical error. Three estimates (with different noise realizations) at the SNR of 85 dB are plotted in red. These are barely distinguishable from the actual room shape. At a lower SNR of 65 dB we observe a considerably larger deviation from the true geometry (there is one particular outlier).

A localization experiment is depicted in Fig. 3b). The source was moving along a lemniscate with the parametric equation

$$(x,y) = \left(1 + \frac{2\cos t}{1 + \sin^2 t}, \frac{3\cos t\sin t}{1 + \sin^2 t}\right),\$$

with $\Delta t = 0.1$ between localizations. As before, the green line shows the noiseless trajectory tracking and is identical to the true



Fig. 3. Numerical simulations with noisy delays. (a) Room geometry estimation, SNR = 65 dB (blue), SNR = 85 dB (red) and SNR = Inf (green). (b) Source tracking: 300 realizations at SNR = 30 dB (blue), 300 realizations at SNR = 40 dB (red), noiseless (green).

trajectory. 300 estimated trajectories at SNR = 40 dB are given in red, and 300 estimates at SNR = 30 dB are given in blue.

6. CONCLUSION

We examined the problem of estimating the geometry of a room from its RIR. We have demonstrated that for many rooms it is possible to reconstruct the room from a single RIR in a unique way. We stated the theorem about the uniqueness of the solution and an algorithm to estimate the room. We also studied how these results may be used in an indoor localization problem and gave an algorithm that performs this localization. Correctness of both algorithms is demonstrated through simulations. Currently, we are considering ways to increase the robustness of the geometry estimation algorithm to measurement noise. Also, we are investigating options for potentially avoiding the combinatorial search.

7. REFERENCES

- M. Kac, "Can one hear the shape of a drum," Am. Math. Mon., vol. 73, pp. 1–23, 1966.
- [2] F. Antonacci, A. Sarti, and S. Tubaro, "Geometric reconstruction of the environment from its response to multiple acoustic emissions," in *Proc. IEEE Int. Conf. Acoust., Speech, and Signal Proc.*, Dallas, 2010, pp. 2822–2825.
- [3] D. Ba, F. Ribeiro, C. Zhang, and D. Florencio, "L1 regularized room modeling with compact microphone arrays," in *Proc. IEEE Int. Conf. Acoust., Speech, and Signal Proc.*, Dallas, 2010, pp. 157–160.
- [4] S. Tervo and T. Korhonen, "Estimation of reflective surfaces from continuous signals," in *Proc. IEEE Int. Conf. Acoust., Speech, and Signal Proc.*, Dallas, 2010, pp. 153–156.
- [5] M. Kuster, D. de Vries, E. M. Hulsebos, and A. Gisolf, "Acoustic imaging in enclosed spaces: Analysis of room geometry modifications on the impulse response," *J. Acoust. Soc. Am.*, vol. 116, no. 4, pp. 2126–2137, Oct. 2004.
- [6] J. B. Allen and D. A. Berkley, "Image method for efficiently simulating small-room acoustics," *J. Acoust. Soc. Am.*, vol. 65, no. 4, pp. 943–950, 1979.
- [7] J. Borish, "Extension of the image model to arbitrary polyhedra," J. Acoust. Soc. Am., vol. 75, no. 6, pp. 1827–1836, 1984.
- [8] E. Lehmann and A. Johansson, "Prediction of energy decay in room impulse responses simulated with an image-source model," J. Acoust. Soc. Am., vol. 124, pp. 269–277, 2008.
- [9] A. Kirmani, T. Hutchison, J. Davis, and R. Raskar, "Looking around the corner using transient imaging," in *Proc. IEEE Int. Conf. on Computer Vision*, 2009, pp. 159–166.