BLIND ESTIMATION AND LOW-RATE SAMPLING OF SPARSE MIMO SYSTEMS WITH COMMON SUPPORT

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ABSTRACT

We present a blind estimation algorithm for multi-input and multi-output (MIMO) systems with sparse common support. Key to the proposed algorithm is a matrix generalization of the classical annihilating filter technique, which allows us to estimate the nonlinear parameters of the channels through an efficient and noniterative procedure. An attractive property of the proposed algorithm is that it only needs the sensor measurements at a narrow frequency band. By exploiting this feature, we can derive efficient sub-Nyquist sampling schemes which significantly reduce the number of samples that need to be retained at each sensor. Numerical simulations verify the accuracy of the proposed estimation algorithm and its robustness in the presence of noise.

***Index Terms*—** Blind channel estimation, MIMO systems, distributed sensing, low-rate sampling, annihilating filters

1. INTRODUCTION

Consider a multiple-input and multiple output (MIMO) system shown in Figure 1, where L sensors take measurements of signals generated by I sources. Connecting the sources to the sensors is a set of LI channels, denoted by their impulse responses $\{h_{i,\ell}(t)\}$. We study two problems in this work:

1. *Blind estimation*: If neither the source signals nor the channel information are known *a priori*, to what extent can one recover these quantities from only the sensor measurements?

2. *Low-rate sampling*: Intuitively, when the number of sensors is much greater than the number of sources (*i.e.*, $L \gg I$), the signals acquired at different sensors are highly correlated. Can one make use of this correlation to reduce the sampling rate at each sensor?

The MIMO setup described above appears in numerous applications. In wireless communication systems, several antennas can send out signals that are received by multiple users or multiple antennas of a single user. The channel states are usually unknown and can change (slowly) over time. To probe the channels, a common approach is to periodically send out known training ("pilot") signals [1, 2]. The success of this channel estimation scheme requires cooperation and accurate synchronization between all the sources and receivers, whose difficulty makes blind estimation a very attractive alternative. The MIMO setup in Figure 1 also models many distributed sensing schemes using wireless sensor networks (*e.g.*, sound acquisition, underwater target tracking, and binaural hearing aids). In these applications, the channels can only be estimated in a blind fashion, as there is no coordination between the sources and receivers. Furthermore, the distributed nature of the sensors and the resulting tight energy budget make it highly desirable to explore low-rate sampling schemes, which can help reduce the amount of data that need to be transmitted through the network.

Blind estimation of MIMO systems has been previously studied in the literature. Existing approaches either exploit statistical priors on the sources (see, *e.g.*, [3]), or impose deterministic constraints on the unknown channels [4, 5]. In this work, we do not assume any knowledge on the source signals, and instead impose constraints on the channels. More specifically, we consider the case that all channels are sparse and have some common support, *i.e.*,

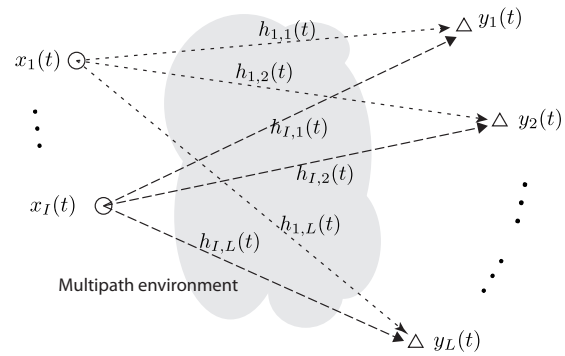

Fig. 1. A MIMO system where I sources and L sensors are linked through a collection of LI channels $\{h_{i,\ell}(t)\}$.

$$h_{i,\ell}(t) = \sum_{k=1}^{K} c_k^{(i,\ell)} \delta(t - t_k), \tag{1}$$

where $\left\{c_k^{(i,\ell)}\right\}_{k=1}^{K}$ are the unknown coefficients of the impulse response linking the ith source to the ℓth sensor, and $\{t_k\}_{k=1}^{K}$ are the unknown *common* support of all the channels.

The sparse common support (SCS) model [6, 2] is a reasonable assumption for many real-world channels. Sparsity is often observed in multipath environments, where each individual path gives rise to an impulse in the channel response function [1, 7]. The common support assumption is relevant when the distances between sensors are much smaller than distance traveled by the electromagnetic (or sound) wave in a time related to the inverse signal bandwidth (see [2] for a more detailed justification). In this case, certain frequency subbands of the channel response functions are well approximated by the SCS model, even though the full channel response functions might not agree with this assumption.

The rest of the paper is organized as follows. After a precise definition of the MIMO system and SCS model in Section 2, we present two main contributions in this paper: Section 3 describes a novel *blind estimation* algorithm based on generalized annihilating filters [8, 9]; The proposed algorithm only requires a small number of frequency samples of the sensor measurements, and therefore naturally leads to a *distributed low-rate sampling* scheme, which we briefly discuss in Section 4. Numerical results in Section 5 verify the effectiveness of the proposed algorithm and its robustness under a wide range of channel noise levels. We conclude the paper in Section 6.

2. PRELIMINARIES

2.1. MIMO System Formulations

Consider a MIMO system with I sources $x_1, x_2, \ldots, x_I$ and L sensors $y_1, y_2, \ldots, y_L$. The signal measured at each sensor is the sum of all source signals going through the corresponding channels, *i.e.*,

$$y_\ell(t) = \sum_{i=1}^{I} h_{i,\ell}(t) * x_i(t), \qquad 1 \le \ell \le L, \tag{2}$$

where $\{h_{i,\ell}(t)\}$ are the channel responses as defined in (1).

We suppose that all the signals are of finite-length and can thus be extended to periodic signals, for some period T. By computing the Fourier series on both sides of (2), we can write the frequency-domain counterpart of (2) in a compact matrix-vector form

$$\boldsymbol{Y}[m] = \boldsymbol{H}[m]\,\boldsymbol{X}[m], \tag{3}$$

where $\boldsymbol{X}[m] \stackrel{\text{def}}{=} \big[X_i[m]\big]_{I\times 1}, \boldsymbol{Y}[m] \stackrel{\text{def}}{=} \big[Y_\ell[m]\big]_{L\times 1}$ and $\boldsymbol{H}[m] = \big[H_{i,\ell}[m]\big]_{L\times I}$[1] denote the mth Fourier coefficients of the source signals, sensor measurements, and channel responses, respectively.

2.2. The SCS Model in the Fourier Domain

In the Fourier domain, the channel impulse responses of the SCS model (1) can be written as

$$H_{i,\ell}[m] = \sum_{k=1}^{K} c_k^{(i,\ell)} u_k^m, \tag{4}$$

where $u_k \stackrel{\text{def}}{=} e^{-j2\pi\omega t_k/T}$. A fundamental property of these sum-of-exponential signals is that they can be "annihilated" by a $(K+1)$-tap filter, *i.e.*, there exist a set of $K+1$ coefficients $\{a_k\}_{0\le k\le K}$ such that

$$\sum_{k=0}^{K} a_k H_{i,\ell}[m-k] = 0, \qquad \text{for all } m. \tag{5}$$

Furthermore, the exponents $\{u_k\}$ are the roots of the polynomial formed by the annihilating coefficients [8, 9], *i.e.*,

$$a_0 x^K + a_1 x^{K-1} + \ldots + a_{K-1}x + a_K = a_0 \prod_{k=1}^{K}(x - u_k). \tag{6}$$

The above expression implies that the annihilating coefficients $\{a_k\}$ are fully determined by the exponents $\{u_k\}$ and are independent of the weights $\left\{c_k^{(i,\ell)}\right\}$ in (4). In the SCS model, all the channel responses have the same support, and therefore their Fourier transforms $H_{i,\ell}[m]$ share the same exponents $\{u_k\}$. It follows that we can generalize the classical annihilating filter in (5) to the following matrix form

$$\sum_{k=0}^{K} a_k \boldsymbol{H}[m-k] = \boldsymbol{0}. \tag{7}$$

This "matrix annihilation" formula captures all the SCS properties in the MIMO system and will play an important role in the proposed blind estimation algorithm described in Section 3.

2.3. Inherent Ambiguities

Given the sensor measurements $\boldsymbol{Y}[m]$ as defined in (3), our goal is to simultaneously estimate the unknown source signals $\boldsymbol{X}[m]$ and the unknown channels $\boldsymbol{H}[m]$, subject to the constraint that the channels $\boldsymbol{H}[m]$ satisfy the SCS model as in (4).

To be clear, it is not possible to fully determine $\boldsymbol{X}[m]$ and $\boldsymbol{H}[m]$ from the sensor measurements $\boldsymbol{Y}[m]$. In fact, one can easily verify the following from our mathematical formulation: If $\{\boldsymbol{X}[m], \boldsymbol{H}[m]\}$ is a solution to (3) with $\boldsymbol{H}[m]$ satisfying (4), then

$$\{\xi^{-m}\boldsymbol{E}^{-1}\boldsymbol{X}[m], \xi^m \boldsymbol{H}[m]\boldsymbol{E}\} \tag{8}$$

is also a valid solution, where $\boldsymbol{E}$ is an arbitrary non-singular constant matrix and $\xi \stackrel{\text{def}}{=} e^{j2\pi\tau/T}$ for some $\tau \in \mathbb{R}$. In the time domain, the phase term ξ^m in (8) points to an inherent ambiguity in time delay: We can always set the sources and channels to $\{x_i(t+\tau), h_{i,\ell}(t-\tau)\}$ for arbitrary τ without changing their convolution results. The matrix $\boldsymbol{E}$ in (8) indicates that we can only reconstruct the coefficients of $\{\mathbf{X}[m], \boldsymbol{H}[m]\}$ up to the linear subspaces they expand.

Finally, we note that the above ambiguities become trivial for single-input and multiple-output (SIMO) systems, as the matrix $\boldsymbol{E}$ degenerates to a scalor. In this case, we aim to reconstruct the unknown source and the channels up to a common time shift and a scalar multiplication.

3. THE PROPOSED BLIND ESTIMATION ALGORITHM

In this section, we present our blind estimation algorithm for sparse MIMO systems with common support. For simplicity of exposition, we first consider the SIMO case, which provides useful insight on how to deal with the unknown multipath channels. We then discuss the generalization to the MIMO case.

3.1. The SIMO Case

In a SIMO system, each output $y_\ell(t)$ is the result of a single source signal $x(t)$ going through the corresponding channel $h_\ell(t)$. It follows that the Fourier domain system equation (3) can be simplified as

$$Y_l[m] = H_\ell[m]\,X[m], \qquad 1 \le \ell \le L, \tag{9}$$

where $\{Y_\ell[m]\}$ are known but $\{H_\ell[m]\}$ and $X[m]$ are unknown. Using the matrix annihilation property of $H_l[m]$'s in (5), we can prove the following result.

Proposition 1 *In a SIMO system with SCS channels, if the number of sensors L is greater than or equal to the cardinality of the channel support K, i.e.,*

$$L \ge K$$

and if there exists a subband of at least $K+3$ continuous Fourier coefficients such that $X[m] \ne 0$ for $m_0 \le m < m_0 + K + 3$, then the system can be fully resolved up to two free parameters, namely an amplitude ambiguity e and a delay ambiguity τ.

Remark: The proposition indicates that a SIMO system can always be fully resolved from the sensor measurements as long as we have enough sensors in the system. The requirement that $X[m] \ne 0$ at $K+3$ consecutive frequency indices are very mild. In fact, it holds with probability one if the source signal $X[m]$ is drawn from any continuous probability distribution.

Proof: For any $m \in [m_0, m_0 + K + 2]$, we can rewrite (9) as

$$H_\ell[m] = Y_\ell[m]/X[m]. \tag{10}$$

On substituting this equality into (5) and defining $b_{k,m} \stackrel{\text{def}}{=} \frac{a_k}{X[m-k]}$, we get

$$\sum_{k=0}^{K} a_k \frac{Y_\ell[m-k]}{X[m-k]} = \sum_{k=0}^{K} Y_\ell[m-k] b_{k,m} = 0. \tag{11}$$

For every fixed m, (11) represents L different linear equations (for $1 \le \ell \le L$) with $K+1$ unknowns. Given $L \ge K$, we can show

[1] Note that $H_{i,\ell}[m]$ lies at the (ℓ, i)-th position of matrix $\boldsymbol{H}[m]$.

that this system of linear homogeneous equations is always solvable, *up to an unknown factor* d_m. It follows that we can obtain $\widetilde{b}_{k,m} \stackrel{\text{def}}{=} d_m b_{k,m} = \frac{a_k d_m}{X[m-k]}$, or equivalently, in a matrix form,

$$\widetilde{\boldsymbol{B}} = (\widetilde{b}_{k,m}) = \begin{bmatrix} \frac{a_0 d_m}{X[m]} & \frac{a_0 d_{m+1}}{X[m+1]} & \frac{a_0 d_{m+2}}{X[m+2]} & \cdots \\ \frac{a_1 d_m}{X[m-1]} & \frac{a_1 d_{m+1}}{X[m]} & \frac{a_1 d_{m+2}}{X[m+1]} & \cdots \\ \frac{a_2 d_m}{X[m-2]} & \frac{a_2 d_{m+1}}{X[m-1]} & \frac{a_2 d_{m+2}}{X[m]} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$= \boldsymbol{\Lambda}_a \begin{bmatrix} \frac{1}{X[m]} & \frac{1}{X[m+1]} & \frac{1}{X[m+2]} & \cdots \\ \frac{1}{X[m-1]} & \frac{1}{X[m]} & \frac{1}{X[m+1]} & \cdots \\ \frac{1}{X[m-2]} & \frac{1}{X[m-1]} & \frac{1}{X[m]} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \boldsymbol{\Lambda}_d, \quad (12)$$

where $\boldsymbol{\Lambda}_a$ and $\boldsymbol{\Lambda}_d$ are diagonal matrices with entries a_k's and d_m's, respectively, and in the middle is a Toeplitz matrix.

Through simple manipulations of the terms in $\widetilde{\boldsymbol{B}}$, we can verify the following relation

$$\frac{\widetilde{b}_{k-1,m}\widetilde{b}_{k+1,m+1}}{\widetilde{b}_{k,m}\widetilde{b}_{k,m+1}} = \frac{a_{k-1}a_{k+1}}{a_k^2} \stackrel{\text{def}}{=} s_k, \quad (13)$$

which, upon setting $a_0 = 1$, can be used to solve for the rest of the a_k's up to one degree of freedom, *i.e.*,

$$a_k = \left(\prod_{j=1}^{k-1} s_j^{k-j} \right) a_1^k. \quad (14)$$

It can be shown that the unknown term a_1^k in the above expression comes from the intrinsic ambiguity of time delay τ, and can be eliminated by simply setting $a_1 = 1$. We omit further details on this due to space constraint.

With the annihilating coefficients $\{a_k\}$ obtained in (14), we can compute the unknown exponents $\{u_k\}$ (and thus the time delay parameters $\{t_k\}$) by factorizing the polynomial in (6). Finally, for fixed $\{u_k\}$, the input-output relation in (3) becomes a set of linear equations. The remaining unknowns (*i.e.*, $\left\{c_k^{(\ell)}\right\}$ and $X[m]$) can then be obtained by inverting this linear system. ■

3.2. Generalizations to the MIMO Case

Now we consider a general MIMO system. In this case, the number of unknowns (including I source signals and LI channels) is much greater than that under the SIMO case (one source signal and L channels). To uniquely determine these parameters, we consider a multi-frame setting, which can be realized by letting the sources send out multiple frames, or more simply, by receiving a long sequence of signals and dividing them into frames on the sensor side. Given J consecutive frames, the relation (3) can be written as

$$\begin{bmatrix} \boldsymbol{Y}^{(1)}[m] & \ldots & \boldsymbol{Y}^{(J)}[m] \end{bmatrix} = \boldsymbol{H}[m] \begin{bmatrix} \boldsymbol{X}^{(1)}[m] & \ldots & \boldsymbol{X}^{(J)}[m] \end{bmatrix}$$

where $\boldsymbol{X}^{(j)}[m] \in \mathbb{R}^I$ and $\boldsymbol{Y}^{(j)}[m] \in \mathbb{R}^L$ are, respectively, the input and output signals at the jth frame.

In what follows, we make a mild assumption that the J vectors $\{\mathbf{X}^{(j)}[m]\}_{1 \le j \le J}$ are "rich" enough so that they span the entire space $\mathbb{R}^I$, *i.e.*,

$$\text{span}\left\{ \boldsymbol{X}^{(1)}[m], \ldots, \boldsymbol{X}^{(J)}[m] \right\} = \mathbb{R}^I. \quad (15)$$

Under this assumption, the matrix $\begin{bmatrix} \mathbf{Y}^{(1)}[m] & \ldots & \mathbf{Y}^{(J)}[m] \end{bmatrix}$ spans the same subspace of $\mathbb{R}^L$ as the range space of the matrix $\boldsymbol{H}[m]$. We can then perform an SVD on $\begin{bmatrix} \mathbf{Y}^{(1)}[m] & \ldots & \mathbf{Y}^{(J)}[m] \end{bmatrix}$ and obtain an L-by-I matrix $\boldsymbol{Z}[m]$ whose columns are orthogonal and span the range space of $\boldsymbol{H}[m]$. It follows that there exists a non-singular $I \times I$ matrix $\boldsymbol{C}[m]$ such that

$$\boldsymbol{Z}[m]\,\boldsymbol{C}[m] = \boldsymbol{H}[m]. \quad (16)$$

We note that the above equality is simply a matrix extension to (10), where $\boldsymbol{Z}[m]$ (as obtained from the SVD of $[\mathbf{Y}^{(1)}[m] \cdots \mathbf{Y}^{(J)}[m]]$) is analogous to $Y_\ell[m]$, and $\boldsymbol{C}[m]$ (an unknown coefficient matrix) is analogous to $1/X_\ell[m]$. Similar techniques to those used in the proof of Proposition 1 can then be employed for solving the MIMO system. Due to space constraint, we merely state the following proposition and leave its proof to [10].

Proposition 2 *In a MIMO system with SCS channels, let I be the number of sources, L the number of sensors and K the cardinality of the channel support. If*

$$L \ge KI,$$

and if there exists a subband of at least $K+3I$ frequency indices such that (15) *holds for $m_0 + 1 \le m < m_0 + K + 3I$, then the MIMO system can be fully resolved up to an amplitude ambiguity matrix $\boldsymbol{E}$ and a delay ambiguity τ.*

4. LOW-RATE SAMPLING SCHEME

We see from the requirements of Propositions 1 and 2 that the proposed blind estimation algorithm only needs a small subband of sensor measurements. Consequently, we can employ a similar approach as used in [1] to derive a distributed low-rate sampling scheme, which is summarized by the following proposition.

Proposition 3 *Under the same condition as stated in Proposition 2, perfect reconstruction on all the sensor measurements can be achieved with probability one, given that we keep $L \ge KI$ sensor samples on a subband of $K + 3I$ frequency indices and $L' \ge I$ sensor samples on all the other frequency indices.*

Remark: This proposition indicates that dense sampling at all the sensors is only required in a limited subband (of $K + 3I$ frequency indices). Beyond this subband, fewer sensor samples are required, and we can still fully reconstruct all the sensor measurements at a central receiver.

Proof: First consider a SIMO system. If we have $K + 3$ consecutive frequency indices of $L \ge I$ sensors, Proposition 1 shows that we can recover all the channel parameters. Given the recovered channel, we only need one of the sensors to work on the other subband to estimate the source signal.

Analogously, in MIMO systems, we can recover all the channel parameters with $L \ge KI$ sensor samples on $K + 3I$ consecutive frequency indices, as shown in Proposition 2. After that, we only need I sensor measurements on each frequency index to uniquely determine the I source signals. ■

5. NUMERICAL EXPERIMENT

In this section we verify the proposed blind estimation algorithm through numerical experiments. In our simulations, we let randomly generated source signals from a white Gaussian distribution to go through the unknown channels, and the retrieved sensor measurements are contaminated by additive white Gaussian noise. The channel delay parameters $\{t_k\}_{k=1}^K$ are uniformly distributed and the amplitude parameters $\left\{c_k^{(i,\ell)}\right\}_{k=1}^K$ have independent Gaussian distributions. Reconstruction results are then directly compared with the

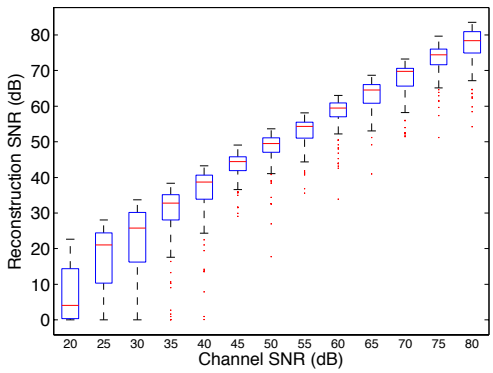

Fig. 2. Reconstruction result of a MIMO system. The experiments are performed 100 times (with different random seeds) at each SNR point. The box plot shows the median (read line), 25th and 75th percentile (blue boxes) and the extreme data points (black lines), respectively.

ground truth. Since the MIMO systems bear an intrinsic subspace ambiguity (see Section 2.3), we compute the reconstruction errors as the distances of the reconstructed signal $\widehat{\boldsymbol{X}}$ to the subspace of all possible $\boldsymbol{X}$'s that explain the output, *i.e.*,

$$\text{Err} = \|\widehat{\boldsymbol{X}} - \boldsymbol{E}\boldsymbol{X}\|_2^2, \quad \text{with } \boldsymbol{E} = \widehat{\boldsymbol{X}}\boldsymbol{X}^\dagger,$$

where $\boldsymbol{X}^\dagger = \boldsymbol{X}^T(\boldsymbol{X}\boldsymbol{X}^T)^{-1}$ is the pseudo-inverse of the matrix $\boldsymbol{X}$.

Figure 2 shows the reconstruction errors with respect to the channel signal-to-noise ratios (SNRs). In our experiment, we use $L = 9$ sensors to sense $I = 2$ sources. The number of frames is equal to $J = 6$, with the length of each frame set to $M = 45$. The common support of the sparse channels contains $K = 4$ impulses. We observe that near perfect reconstruction can be achieved at relatively high channel SNR regimes (50 dB and above). At lower SNR levels (around 25–35 dB), the algorithm still provides stable and accurate estimates.

The noise robustness of our reconstruction algorithm benefits from the multi-frame setup of the system. In general, the more frames we can use, the better recovery result we get. On the other hand, we found that the loss of accuracy by reducing the number of frames can be effectively compensated if we apply a local optimization algorithm, which can significant enhances the performance even when only one frame is available. Starting from the estimations obtained from the proposed algorithm, we apply local iterative optimization to refine the parameters $\{u_k\}$ (and equivalently, $\{t_k\}$) so that they are better fits for the observation model (3) and SCS constraint (4). Figure 3 shows the estimation results on a SIMO system by using a single frame. We can see that the improvement brought by the local optimization (refinement) is substantial, and the resulting reconstruction performance is even comparable to that obtained by using the ground truth $\{t_k\}$.

6. CONCLUSION

We presented a novel algorithm for estimating MIMO systems with sparse common support. Based on a matrix generalization of the annihilating filter technique, the proposed algorithm is able to blindly estimate the unknown source signals and the channel information by using only the sensor measurements. A useful property of the proposed algorithm is that it only needs sensor measurements on a narrow frequency band. By exploiting this property, we derived an efficient low-rate sampling scheme, which can significantly reduce the number of samples that need to be retained at each sensor. Numerical experiments verify that the proposed algorithm can achieve perfect reconstruction in the noise-free case and can obtain stable and accurate estimations in the presence of modest noise.

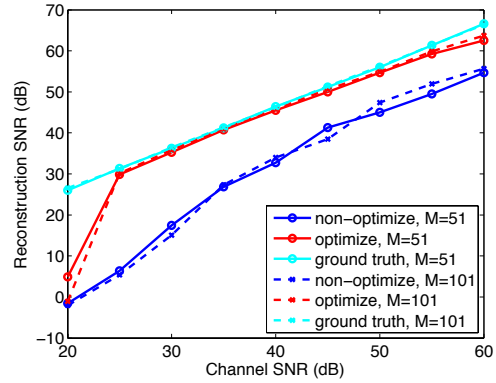

Fig. 3. Reconstruction result of a SIMO system. This figure shows the reconstruction SNRs of three different approaches. The first (blue, bottom) uses the estimated t_k directly from the output of the proposed algorithm; the second approach (red, middle) uses the optimized parameters t_k as described in the text; and the third approach (cyan, top) uses the (unknown) ground truth t_k. M denotes the number of frequency points used for reconstruction.

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